Seventh-grade students problem posing from open-ended situations

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The present study was the final phase of a three-year project in which problem-posing programs were developed for the third, fifth, and seventh grades. The aims of the study were as follows:

1. to trace the development of seventh-grade students' problem posing across a range of mathematical situations;

2. to trace the developments of individual children as they participate in a 3-month classroom problem-posing program;

3. to monitor changes in children's perceptions of, and attitudes towards, problem posing and problem solving;

4. to identify links between students' problem-posing and problem-solving abilities.

Background

Problem posing is recognized as a significant component of the mathematics curriculum and is considered to lie at the heart of mathematical activity (e.g., Brown & Walter, 1993; Moses, Bjork, & Goldenberg, 1990; Silver & Cai, 1996). The inclusion of activities in which students generate their own problems, in addition to solving pre-formulated examples, has been strongly recommended by several national bodies (e.g., Australian Association of Mathematics Teachers, 1996; National Council of Teachers of Mathematics, USA, 1989; Streefland, 1993). Despite its significance, problem posing has not received the attention it warrants from mathematics education researchers. We know comparatively little about children's abilities to create their own problems in different mathematical contexts, about the processes they use, and about the extent to which these abilities are linked to their competence in problem solving. There is also insufficient information on how children respond to programs designed to develop their problem posing (Silver, 1994). This situation needs to be redressed, given that problem-posing activities in the curriculum can foster more diverse and flexible thinking, enhance students' problem-solving skills, broaden their perceptions of mathematics, and enrich and consolidate basic concepts. Student-generated problems can also provide us with important insights into children's understanding of mathematical concepts and processes, as well as their perceptions of, and attitudes towards, problem solving and mathematics in general (Brown & Walter, 1993; English, in press c; English, Cudmore, & Tilley, in press; Silver, 1994; Van den Heuvel-Panhuizen, Middleton, & Streefland, 1995).

The lack of research on curricula designed to foster children's problem posing means there are few theoretical and practical frameworks to guide program development (Silver, 1994). Building on my previous studies (English, in press a, in press b) and on related literature (e.g., Brown & Walter, 1993; Cobb & Bauersfeld, 1995; Schoenfeld, 1992; Silver & Cai, 1996) I developed the theoretical framework outlined in Figure 1.

This framework guided the construction of the problem-posing program, described in a subsequent section. As it is not possible to address the entire framework, I consider only those elements that are fundamental to the findings examined here. These findings are concerned with children's problem creations from two types of open-ended situations, namely, descriptive problem stories, and open symbolic expressions. Posing problems from these situations primarily requires an understanding of, and facility with, problem structures, and an ability to think diversely and flexibly.

Theoretical Framework

Understanding Problem Structures

One of the fundamental elements of problem posing is understanding just what a problem is (Brown & Walter, 1993). This includes being able to recognise its underlying structure and to detect corresponding structures in related problems. Structure may be defined as "form abstracted from its linguistic expression" (Freudenthal, 1991, p. 20). While not denying the importance of problem context (Freudenthal, 1991), children need

KEY ELEMENTS OF PROBLEM POSING

Knowledge and Reasoning

- Understanding problem structures and recognising related structures;
- Knowing problem design, in particular, recognising critical units of information;
- Being able to model and transform given structures, as well as build new ones;
- Knowing whether and how a posed structure constitutes a solvable problem;
- Being able to think in mathematically diverse ways;
- Knowing how and when to apply processes of analogical reasoning;
- Being able to reason critically in assessing problems and problem experiences.

Metacognitive Processes

- Communicating one's perceptions of, and preferences for, different problem types;
- Reviewing and enhancing self-efficacy expectations;
- Improving one's disposition towards problem posing and problem solving.

Sociological Factors

- Participating in classroom communities of philosophical and mathematical inquiry;
- Engaging in constructive dialogue and debate;
- Sharing and critiquing problem creations.

Figure 1 Key Elements of Problem Posing

to recognise the mathematical structures of problem situations if they are to utilise these to generate new examples and questions; this requires them to place the contextual features in the background and bring the structural elements to the fore. That is, children need to construct meaningful mental models or representations that recognise the important mathematical ideas and how they are related. The importance of such mental models in mathematical learning has been well documented (e.g., English, in press c; English & Halford, 1995; Novick, 1995; Silver, 1981). Indeed, a lack of such models has been shown to be responsible largely for the difficulties children experience with operational word problems, especially those with complex structures. For example, problems in which there is not a clear mapping between the problem situation and the operation required for solution cause particular difficulties (e.g., Sally has saved $24. This is 3 times the amount Peter has saved. How much has Peter saved?; English in press c; Nathan, Kintsch, & Young, 1992; Stern, 1993).

The complexity of problem structure is also determined, in part, by its linguistic or syntactic properties (Mayer, Lewis, & Hegarty, 1992; Silver & Cai, 1996). Mayer et al. found that problem-solving difficulty seemed to be related to linguistic complexity, with problems containing assignment propositions easier than those with relational or conditional propositions. For example, a problem that asks, "How much did the lunch cost?" would be easier than one which asks, "How much more does the bike cost than the scooter?" or, "How much would the drinks cost if each person was only allowed two cartons of juice?" The nature and number of distinct semantic relations embodied in a problem also have a bearing on its complexity (Marshall, 1995; Silver & Cai, 1996). For example, a story problem that involves both multiplication and subtraction would be more complex than a comparable case involving only one of these.

Another important factor in children's facility with problem structure is their awareness of problem design. In generating their own problems, children must recognise the critical items of information that are required for problem solution (in contrast to the other items such as contextual information). This awareness of design includes recognising the nature and role of the "known" and "unknown" information entailed in their posed problem, as well as any constraints placed on goal attainment (Moses et al., 1993). This knowledge is necessary for determining whether and how a posed problem structure constitutes a solvable problem, a basic element of problem posing (Brown & Walter, 1993).

Thinking in Mathematically Diverse Ways

Being able to perceive mathematical situations in diverse ways is not only fundamental to children's problem-posing development but also to their overall mathematical growth (English & Halford, 1995; NCTM, 1989; Smith & Silver, 1995). Interpreting a mathematical situation in more than one way is particularly important in children's understanding of operational situations and their ability to generate new operational problems. Although there exists substantial literature on children's abilities to *solve* operational problems (e.g., Bergeron & Herscovics, 1990; Carpenter et al., 1993; Fuson, 1992; Greer, 1992), there is little information on children's abilities to *pose* them. We are thus left with an incomplete account of children's numerical facility. Part of this facility requires being able to assign several meanings to the formal symbols (+, -, x, ∏). Unfortunately, children's school experiences rarely provide them the opportunity to interpret these symbols in a variety of ways; the meanings usually assigned to them are those elementary concepts children are taught first (Fischbein, Deri, Nello, & Marino, 1985; Fuson, 1992; Stern, 1995). This appeared to be the case in recent research (English, in press a) where grade 3 children were found to be inflexible in their problem creation, experiencing considerable difficulty in recognising formal symbolism as representing a range of problem situations (e.g., they would continue to pose only a "take-away" problem for 9-6=3). Broadening children's perceptions of mathematical situations thus appears a major area in need of attention in children's problem-posing development.

Program Development and Implementation

As described in this section, a 3-month problem-posing program was developed for several classes of seventh-grade students. Given the confines of the students' existing curricula, it was not possible to implement problem posing as an integral learning process across the entire mathematics curriculum, although this certainly would have been desirable. Nevertheless, as Wittmann (1995) emphasised, research centred on carefully designed and empirically studied teaching units that are based on fundamental theoretical principles, makes a major contribution to mathematics education. Indeed, Wittmann maintained that "such units are the most efficient carriers of innovation and well-suited to bridge the gap between theory and practice" (p. 369).

The present program represents a refinement of the third- and fifth-grade programs (English, in press a, in press b). The development of these programs reflects elements of Freudenthal's (1991) "thought experiment," which involves envisioning how the teaching-learning process will proceed prior to implementing activities in the classroom. After implementation, one tries to find evidence that indicates whether or not the expectations were realised. The feedback of the practical experience into new thought experiments generates a cycle of curriculum development and research. This cycle has continued throughout the three years of the problem-posing project.

Participants and Selection Procedures

Three classes of seventh-grade students from three state schools participated in the present study, which was conducted throughout 1996. Twenty-three students from across the three classes were chosen for in-depth observation and analysis (mean age of 11. 9 years in term 1). The 23 children (along with an additional six children serving as a small control group) were chosen on the basis of their responses to tests of number sense and novel problem solving; these were administered during the first term of the school year. The tests were modelled on examples that had been used successfully in the previous studies (English, in press a, in press b). The number sense test focused on facility with number and routine computational problem solving, while the novel problem-solving test included examples requiring a range of reasoning processes (e.g., deductive, combinatorial, spatial reasoning), as well as general strategies.

The selected children displayed the following profiles of achievement: (1) strong in number sense but not so in novel problem solving ("SNS" profile; N=6); (2) not strong in number sense but strong in novel problem solving ("SNP;" N=5); (3) strong in both domains ("SB;" N=7); (4) average achievement in both domains ("AB;" N=5). The intention was to include children from the first three profiles only, however difficulty in obtaining sufficient numbers necessitated adding the last category (which, as the results show, proved beneficial).

The 29 children (including the small control group) were individually administered a comprehensive set of problem-posing activities during the second term (prior to the program) and a parallel set towards the end of the fourth term (after the program). The problem-posing program was conducted during the third and fourth terms and comprised 12 weeks of classroom activities (approximately 1.5 hours per week).

Program Design

The program encompassed six main activity types that addressed the problem-posing elements displayed in Fig. 1. Briefly, these activity types are as follows:

Problem exploration and reflection: This involved group and class discussions on the children's perceptions of problem solving and posing, their attitudes towards these, the different approaches they adopt in problem situations, their problem experiences out of class, how problem solving could be made more interesting for them in the classroom, and other related issues. Included here were group debates on issues such as, "You learn more from creating and solving your own problems than from solving ones the teacher makes up."

Problem preferences: Here, the children examined a range of routine and non-routine problems and discussed which ones they would most/least like to solve. They also indicated how they would modify their disliked problems to make them more appealing.

Problem sorting: This involved the children in sorting sets of routine and non-routine problem cards according to similarity in problem structure. The problems were designed such that those with parallel structures had different contexts, and those with different structures were set in the same context.

Modelling new problems on existing structures: The children compared routine and non-routine problems and talked about the important elements in problem design. These included elements such as the nature of the known information, the type of information that is not known, and any constraints placed on goal attainment. After solving some of these problems, the children created their own examples by modelling them on the existing problem structures (analogical reasoning processes play an important role here, as discussed in English, 1997). Diversity of problem context was encouraged in the children's creations.

Creating a new problem from problem components: Open-ended situations: A range of activities was implemented here. These included posing problems when one is given "knowns" only, constraints only, or knowns and constraints (Brown & Walter, 1993). Symbolic examples of these activities included creating problems from open expressions and statements, such as "135-68," "24 x 5," "The answer is 4/5. What is the question?" "A new table costs $567. Matching chairs cost $68 each and arm rests cost an extra $25 per chair," and "Create a division problem where the remainder must be used."

The open-ended descriptive situations included a number of examples that were taken from travel brochures, newspaper items, historical documents etc. One of these is addressed in the next section.

Transforming a given problem into a new problem: In these activities, the children explored ways of creating new problems by modifying the structures of existing problems. This included reversing knowns and unknowns, removing constraints, adding more knowns and/or constraints, and using a "what-if-not strategy" (Brown & Walter, 1993).

An important component of all the children's problem posing activities was their sharing and critiquing of each other's problems. A special critique form was created for this, which the children completed and returned to the author of the problem (English et al., in press). The form asks the problem "reviewer" to consider a number of points including whether the problem is solvable, whether it is challenging and interesting, aspects that the reviewer likes/dislikes, and suggestions on how the problem could be modified (if necessary) and extended.

Throughout the implementation of the program, we tried to establish a community of inquiry involving meaningful dialogue or "connected talking" among the children and teacher (English, in press c; Yackel, 1995). At certain points during the program, and on its conclusion, we asked the children and their teachers to reflect on their experiences and also to comment on specific aspects (e.g., what they liked, did not like, whether they felt their problem-posing and problem-solving skills had improved, how we could improve the program for future classes etc.). All children maintained journals of their problem creations and reflections, and the responses of the selected children were video- and audio-taped.

Children s Responses to One Activity Type: Open-ended Situations

Given the limitations of space, children's responses to one activity type only are reported here. We consider their responses to posing problems from (i) open-ended descriptive situations, as shown in Fig. 2, and (ii) open symbolic expressions (subtraction and multiplication, of the form, 134 - 29 and 35 x 5). The example in Fig. 2 (see page 8) appeared on the post-program activities, with a parallel example included in the pre-program activities. The children were required to construct three different problems from the given information. The open symbolic expressions listed above appeared on the post-program activities, along with other similar examples (parallel cases were presented in the pre-program activities). The children were asked to create two different problems for each symbolic expression. As previously noted, the children explored a number of open-ended situations during the program.

SPOOKY TRAVEL

A 5-day tour of the ghost castles on No Man's Island, departing from Munster Town, costs $1776 per person. A 4-day tour of the bat caves on No Man's Island costs $1400 per person. Departure from Cape Fear to No Man's Island costs $350 less per person. The cost of food for each of the trips is $450 per person if there is just one person travelling, and $400 per person if two or more people are travelling together.

Figure 2 An Open-ended Descriptive Situation

To obtain an idea of the children's progress with these open-ended examples, we consider their pre- and post-program responses and also include a couple of their program responses. The problems they created on the pre- and post-program activities were analysed using the following coding scheme, which draws upon some of the ideas of Silver and Cai, 1996.

Problem creation and solvability

This was concerned with: (i) whether a mathematical problem was created, and (ii) whether the problem was solvable with a unique solution (although problems with more than one solution are important in the curriculum, such problems in the present context reflected a problem design weakness).

Problem complexity (for the descriptive situations)

This focused on: (i) the extent of critical information units included in the problem, (ii) the number of distinct semantic relations, (iii) the number of steps required for solution, and (iv) the type of question posed (assignment, relational, conditional). A critical information unit, as used here, refers to an item of information that is necessary for problem solution. For example, reference to the point of departure in the example in Fig. 2 is a critical information unit, as is a statement on whether food is required. An assignment question addresses one variable, such as, "How much did the trip cost?" while a relational question compares two variables, such as, "How much more does it cost to go on the 5-day tour than the 4-day tour?" (Mayer et al., 1992). A conditional question imposes a constraint, such as, "How much would you have to pay if you wanted to depart from Munster Town and if you wanted to take a friend with you?"

Problem complexity and diversity (for the symbolic expressions)

Children's creations for the symbolic expressions were classified as either basic or complex, with respect to the mathematical structure of the problem (cf. classification system used by Leung, in press). Basic problems were the elementary change and part/part/whole examples for subtraction (e.g., Peter had 9 marbles. He lost 4 marbles. How many marbles does he have now?; Sally has 9 cars. 3 are red and the rest are blue. How many are blue?) and equivalent set problems for the multiplication (e.g., Four children have 3 balloons each. How many do they have altogether?). Complex problems reflected a broadening of the children's thinking and included the more difficult cases such as: (i) comparison and equalise situations for subtraction (e.g., Sally has 6 goldfish. This is 3 more goldfish than Samantha has. How many goldfish does Samantha have?; Sue has 9 marbles. Jenny has 6 marbles. How many more must Jenny win to have as many as Sue?), (ii) scalar or multiplicative comparison problems (e.g., John has 24 cars. Penny has 3 times as many cars as John. How many cars has she?), and (iii) Cartesian product (combinatorial) problems (e.g., Sue has 3 different blouses and 4 different skirts. How many different outfits can she make?).

Children were considered to show diversity if the two problems they created for each expression had different structures, such as one change problem and one comparison problem, or one comparison problem and one equalise problem, or one equivalent set and one multiplicative comparison.

Overview of the Findings for the Descriptive Situations

The children showed a distinct improvement in their abilities to generate problems from open-ended descriptive situations. On the pre-program example, there were two instances of a non-mathematical problem being generated (both from children in the AB profile) and 17 instances of an insolvable problem. Children from the SNP and AB profiles had the greatest difficulty in creating a solvable problem on the pre-program activity, while children from the SB profile were the most competent. On the post-program activity however, every child was able to create a solvable problem, with children in the SNS and SB profiles better able to create problems with unique solutions than children in the remaining profiles. In contrast, the six non-participants ("control group") had difficulty in generating a solvable problem, with 45% of their creations being either non-solvable or a non-mathematical problem.

Developments in the complexity and sophistication of the children's problems between the pre- and post-program activities can be seen in Tables 1 and 2. We examine Table 1 first.

Table 1 Frequencies of Use of Critical Units (CU) and Question Types (QT) by Achievement Profile

	First Problem Created						Second Problem Created							Third Problem Created						
	No. of CU			QT			No. of CU				QT			No. of CU				QT		
Profile	1-2	3-4	5-7	A*	R	C	0	1-2	3-4	5-7	A	R	C	0	1-2	3-4	5-7	A	R	C
Pre-program Problems Generated																				
SNS**	4	2		3	1	2		4	2			1	5			6		1		5
SNP	5			2		2		4	1		1		4		4	1		1		4
SB	1	5	1	4		3		1	5	1	4	2	2		1	5	1	5	1	2
AB	5			2		3	1	4			2		2	1	3	1		1		3
Post-program Problems Generated																				
SNS		4	2	1		5			4	2	2	1	4			3	3		4	2
SNP	1	4			1	4		2	2	1	1	1	3			5				5
SB		3	4	2		5			5	2	4		3		1	4	2		3	4
AB	1	3	1	2		3		2	2	1	3		2			4	1			5

Note. * A: assignment R: relational C: conditional ** SNS: strong in number sense only (N=6); SNP: strong in novel problem solving only (N=5); SB: strong in both (N=7); AB: average in both (N=5). On the pre-program activity, there were 3 cases where a question type could not be assigned and 2 instances in which the question type was both relational and conditional. On the post-program activity, there was one of the latter instances.

Among the more noticeable developments evident in Table 1 was an increase in the number of critical information units the children included in their problems (reflecting an increase in solvable problems). Children from the SNP and AB profiles in particular, showed substantial growth, as was evident in Nathan's (SNP) case. He progressed from being unable to generate a solvable problem prior to the program to creating this problem after the program: *Which would cost more? Being a single person and leaving from Munster Town or having two people leave from Cape Fear to go to No Man s Island?* While children from the SNS profile also showed considerable improvement in their inclusion of critical information units, those from the highest profile (SB) showed little change between the pre- and post-program activity. These children had few difficulties in generating problems prior to the program and were able to create quite sophisticated examples during the program. For example, Adam posed this problem after examining one of the travel brochures: *I ve taken a leap year off work and decided to go on as many holidays as possible. Each time I go on a holiday I have to take the time of the previous holiday to recover for the one coming up. If two holidays go for the same amount of time I ll go on the most expensive one, then the cheapest, then I ll go on another expensive one, then a cheaper one, and so on. What would be the average cost per day whether I m at home recovering or on holiday? P. S. Money is not a concern.*

The program made little difference to the children's posing of relational questions (see Table 1). These were clearly not favored, reflecting the documented difficulties children experience with comparison problems (e.g., Stern, 1993). On the other hand, 59% of all the children's questions were of a conditional type and 35% were assignment questions. This is in contrast to Silver and Cai's (1996) findings where only 5% of their sixth- and seventh-grade students posed conditional questions. Interestingly, it was the SNP children who tended to favour conditional questions on both activities. This reflects one of the findings from the fifth-grade study where children in this category tended to create problems that displayed structural complexity but operational simplicity (English, in press a).

Table 2 shows quite substantial shifts in the children's use of semantic relations and in the complexity of their problem solutions.

Table 2 Frequencies of Use of Semantic Relations (SR) and Solution Steps by Achievement Profile

First Problem Created

Profile	No. of SR 0	1	2	3	No. of Steps 0	1	2	3	4	5>
Pre-program Problems Created										
SNS*	1	4	1		1	4	1			
SNP	3	1	1		3	1	1			
SB		4	3			4	3			
AB	2	3			2	3				
Post-program Problems Created										
SNS		2	3	1		1		1	2	2
SNP		2	3			2	2		1	
SB		1	4	2			3	2		2
AB		4		**		4				

Second Problem Created

Profile	No. of SR 0	1	2	3	No. of Steps 0	1	2	3	4	5>
Pre-program Problems Created										
SNS*	2	1	2	1	2	1	2	1		
SNP	2	2	1		2	2	1			
SB	1	4	1	1	1	4	1	1		
AB	1	4			1	4				
Post-program Problems Created										
SNS		4	2			1	1	4		
SNP		1	4			1	4			
SB		2	5			2	3	1	1	
AB		2		3		2		2		1

Third Problem Created

Profile	No. of SR 0	1	2	3	No. of Steps 0	1	2	3	4	5>
Pre-program Problems Created										
SNS*		3	2	1		3	2	1		
SNP	3	2			3	2				
SB		3	4			2	4		1	
AB	3	1	1		3	1	1			
Post-program Problems Created										
SNS		1	4	1		1	1	1	2	1
SNP		1	4			1	2	2		
SB		2	3	2		1	2	2	2	
AB			5				4			1

Note. SNS: strong in number sense only (N=6); SNP: strong in novel problem solving only (N=5); SB: strong in both (N=7); AB: average in both (N=5). ** One child from the AB profile created a combinatorial problem for her first problem.

As shown in Table 2, the SNP children demonstrated the greatest improvement, especially in their ability to incorporate several semantic relations in their problem; the AB children also showed marked gains. For example, they created problems which incorporated addition, subtraction, and multiplication. Children in the remaining two profiles displayed a noticeable increase in the computational complexity of their problems, with 59% of their post-program problems involving 3 or more steps (in contrast to only 10% previously).

Overview of the Findings for the Symbolic Expressions

The children had little difficulty in actually posing a problem for the subtraction and multiplication expressions. However prior to participating in the program, few children created problems that were structurally complex or diverse, as can be seen in Table 3 (see page 13). It is interesting to note though, that the children who were strong in novel problem solving (SNP and SB profiles) showed the greatest complexity and diversity in their problem creations, prior to the program (this again reflects findings from the previous studies [English, in press a, in press b]).

Children in each profile showed considerable improvement after participating in the program, with their problems displaying greater structural complexity and diversity (in contrast to the control group, where only two could create a complex subtraction or multiplication problem, and only two showed diversity). The AB children in particular, seemed to make substantial progress, just as they did on the previous activity. For example, they created a proportionally greater number of complex subtraction problems than all the other children and also showed greater diversity than the SB children. We would have expected the SB children to show greater development in their problem posing with symbolic expressions. Although they did pose a proportionally greater number of complex multiplication problems than the remaining children, this was not the case with their subtraction problems where they also displayed limited diversity.

It is also worth noting the observed improvement in the children's use of context in their problems. Prior to the program, the children used simple contexts such as, *A farmer had 134 cows. 29 died. How many were left?* After the program, more diverse contexts appeared such as, *At the Channel 7 TV station, 134 fan letters were received every day. Channel 10 received 29 fan letters less than Channel 7 each day. How many fan letters does Channel 10 receive each day?*

Table 3 Proportions of Children Who Created (a) 1 or 2 Complex Problems (b) Diverse Problems for the Symbolic Expressions, by Achievement Profile

		No. of Complex Problems Created				Created Diverse Problems	
		Subtraction		Multiplication		Subtraction	Multiplication
Profile		1	2	1	2		
SNS*	Pre-program	.17	.17			.33	
	Post-program	.5	.17	.33	.33	.83	.5
SNP	Pre-program	.4		.2	.2	.4	.2
	Post-program	.4	.2	.4	.2	.8	.4
SB	Pre-program	.14	.29	.14	.14	.43	.14
	Post-program	.14	.43	.43	.29	.43	.58
AB	Pre-program		.2				.2
	Post-program	.4	.4	.2	.4	.6	.6

Note.* SNS: strong in number sense only (N=6); SNP: strong in novel problem solving only (N=5); SB: strong in both (N=7); AB: average in both (N=5).

Conclusions and Implications

The findings from this study and the previous two studies suggest a number of important implications for implementing problem posing in the primary school. First, it seems that the program described here was a successful learning experience for the seventh-grade students and their teachers. For example, a typical comment from the children's reflections on the program was, *I understand problems heaps more now... I actually got involved and didn t sit back and be lazy, it s more fun and challenging than when I started.* Similarly, one of the grade 7 teachers commented: *Personally I really enjoyed it. I thought it was a way that I had never ever considered approaching problem solving, and I personally did learn a lot from it. I think it freed up the entire class and they really enjoyed the activities. . . they were actually asked what their opinions were; they were asked their thoughts, their feelings, and I think they gave those quite freely.* Although the present program was refined from the previous studies, there is nevertheless considerable room for improvement both in its design and in its implementation. For example, one area in need of attention is children's conceptions of operational situations. The present program needed to devote more time to enriching and broadening the children's understandings; the elementary operational concepts were still very much entrenched (cf. Fischbein et al., 1985).

Although the present sample is small and the data are limited, the responses of children in the different profiles suggest further implications for classroom practice. The SNP and AB children, both of whom were not strong in the number domain, made substantial gains from participating in the program. In particular, the SNP children showed a good deal of divergence in their thinking and in their problem creations, as was found in the previous studies. Their weaknesses in the number domain did not appear to hinder them in their problem creations. In fact, the classroom observations showed that they felt a sense of empowerment in handling mathematical problem situations and delighted in creating challenging and "unusual" problems for their peers to try. Because such children lack numerical proficiency, it is easy to overlook their other mathematical abilities in a number-dominated curriculum. We need to broaden children's mathematical experiences so that all children have the opportunity to reach their maximum development in different mathematical domains (cf., Vygotsky, 1962). We might then see substantial improvement in children's overall mathematical growth.

There is also the need to explore further the links between children's problem-posing and problem-solving abilities and to consider these in designing classroom experiences. The three studies collectively suggest that competence in solving routine computational problems is associated with the posing of computationally complex, but not necessarily structurally complex, problems. Competence in nonroutine problem solving appears associated with the posing of more divergent and structurally complex problems. We need to engage students in a range of problem-posing activities that draw their attention to both the computational and structural complexities of the problems they create and solve. At the same time, they need to be aware of the contextual components and how these can both hinder and enhance problem creation and solution.

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